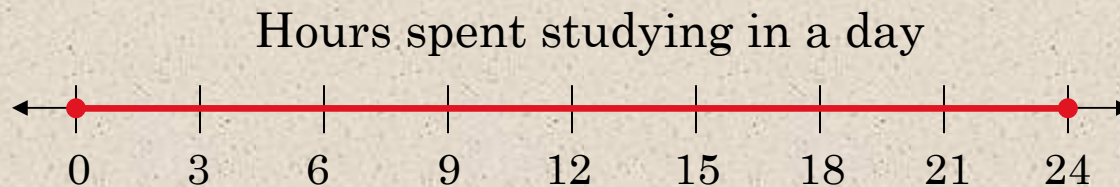Chapter 5 Review

Normal Probability Distributions

Properties of Normal Distributions

A **continuous random variable** has an infinite number of possible values that can be represented by an interval on the number line.

Hours spent studying in a day

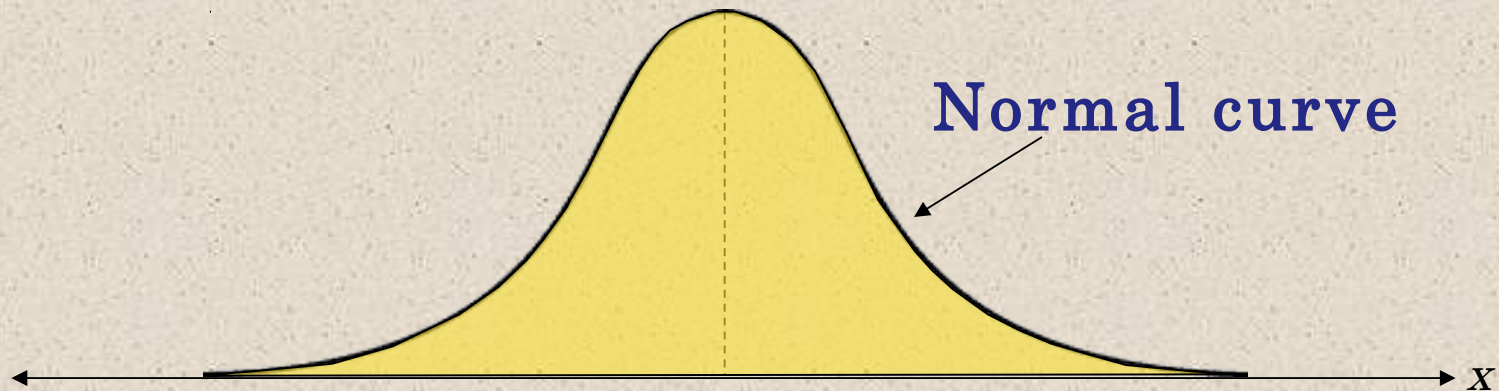0   3   6   9   12   15   18   21   24

The time spent studying can be any number between 0 and 24.

The probability distribution of a continuous random variable is called a **continuous probability distribution.**

Properties of Normal Distributions

The most important probability distribution in statistics is the **normal distribution**.

Normal curve

A normal distribution is a continuous probability distribution for a random variable, x. The graph of a normal distribution is called the **normal curve**.

Properties of Normal Distributions

Properties of a Normal Distribution

1. The mean, median, and mode are equal.

2. The normal curve is bell-shaped and symmetric about the mean.

3. The total area under the curve is equal to one.

4. The normal curve approaches, but never touches the x-axis as it extends farther and farther away from the mean.

5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the *inflection points*.

Questions about the Normal Curve Characteristics

1. What is the area under the curve equal to?

2. What and where are the inflection points?

3. Is the mean greater than the mode and median. Explain.

Properties of Normal Distributions

Inflection points

Total area = 1

$$\mu - 3\sigma \qquad \mu - 2\sigma \qquad \mu - \sigma \qquad \mu \qquad \mu + \sigma \qquad \mu + 2\sigma \qquad \mu + 3\sigma$$
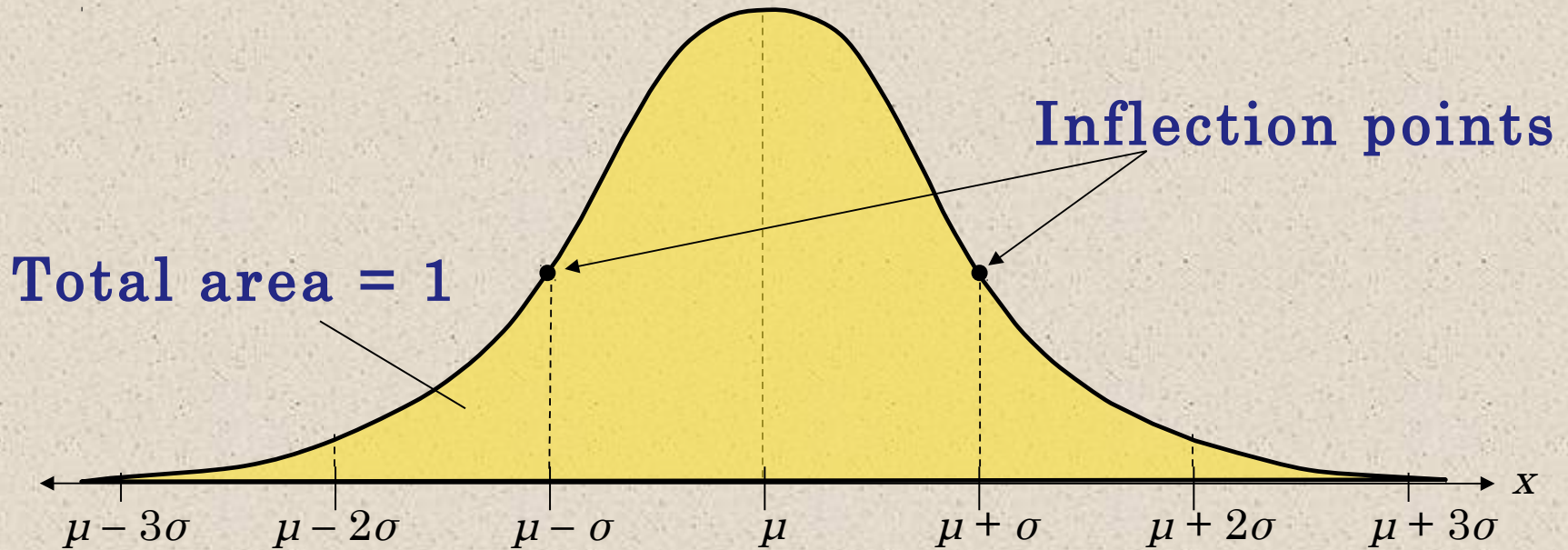
If x is a continuous random variable having a normal distribution with mean μ and standard deviation σ, you can graph a normal curve with the equation

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}. \qquad e \approx 2.178 \qquad \pi = 3.14$$

How to draw The Standard Normal Distribution

The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

The horizontal scale corresponds to *z*-scores.

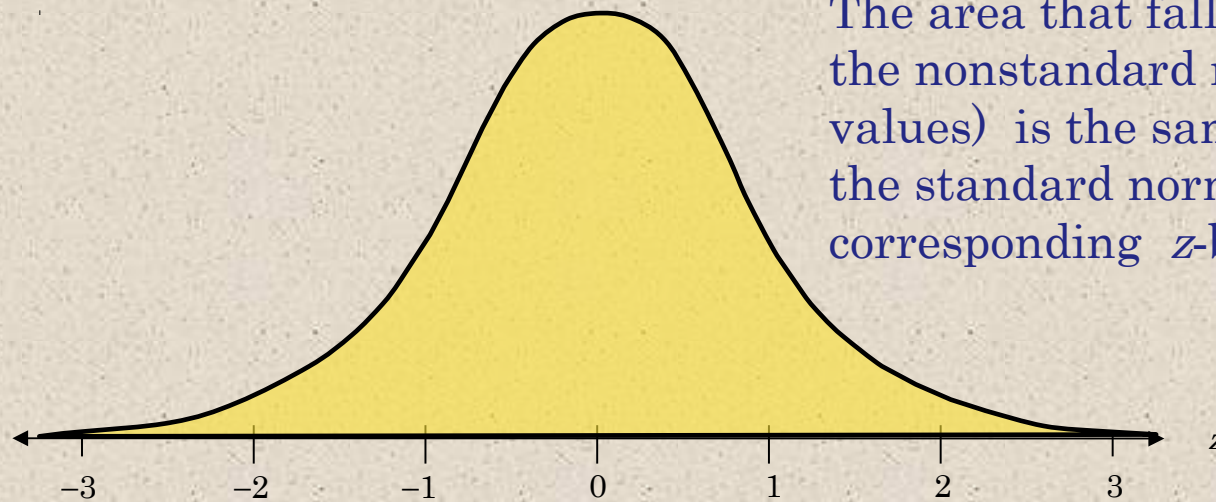$$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \qquad z$$
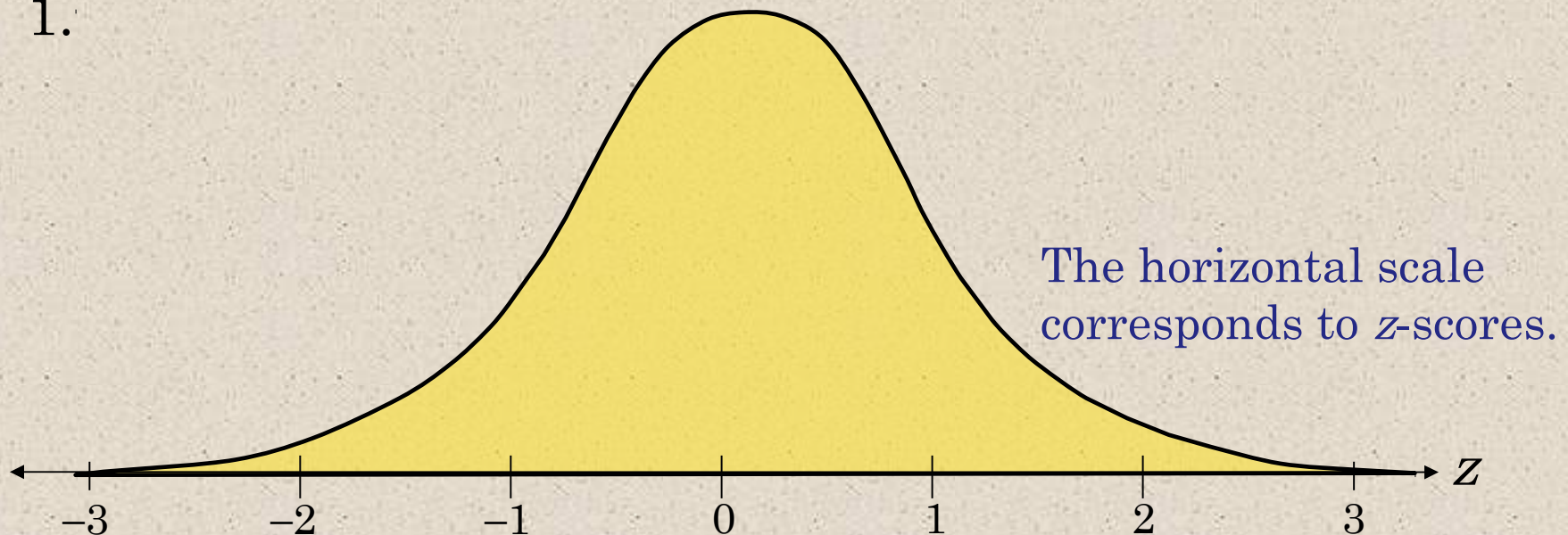
Any value can be transformed into a *z*-score by using the formula $z = \dfrac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \dfrac{x - \mu}{\sigma}$.

The Standard Normal Distribution

If each data value of a normally distributed random variable x is transformed into a z-score, the result will be the standard normal distribution.


The area that falls in the interval under the nonstandard normal curve (the x-values) is the same as the area under the standard normal curve (within the corresponding z-boundaries).

After the formula is used to transform an x-value into a z-score, the Standard Normal Table in Appendix B is used to find the cumulative area under the curve.

The Standard Normal Table

Properties of the Standard Normal Distribution

1. The cumulative area is close to 0 for z-scores close to $z = -3.49$.

2. The cumulative area increases as the z-scores increase.

3. The cumulative area for $z = 0$ is 0.5000.

4. The cumulative area is close to 1 for z-scores close to $z = 3.49$
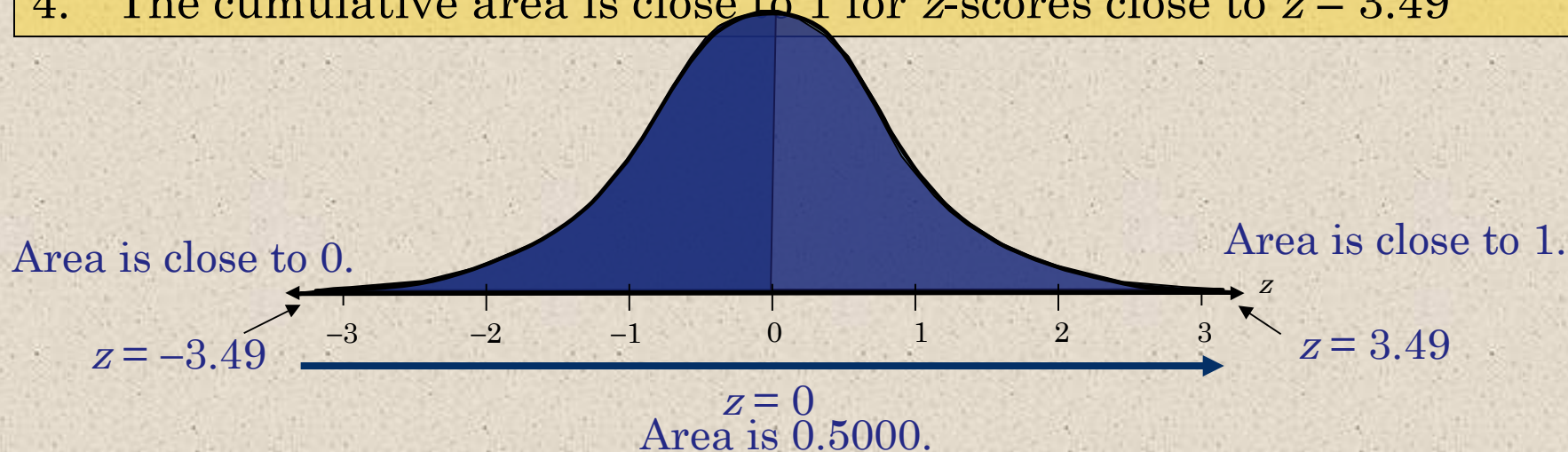
Area is close to 0.

Area is close to 1.

$z = -3.49$

$z = 3.49$

$z = 0$
Area is 0.5000.

The Standard Normal Table

Example:
Find the cumulative area that corresponds to a *z*-score of 2.71.

↓ Appendix B: Standard Normal Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981

Find the area by finding 2.7 in the left hand column, and then moving across the row to the column under 0.01.

The area to the left of $z = 2.71$ is 0.9966.

The Standard Normal Table

Example:
Find the cumulative area that corresponds to a *z*-score of −0.25.

Appendix B: Standard Normal Table

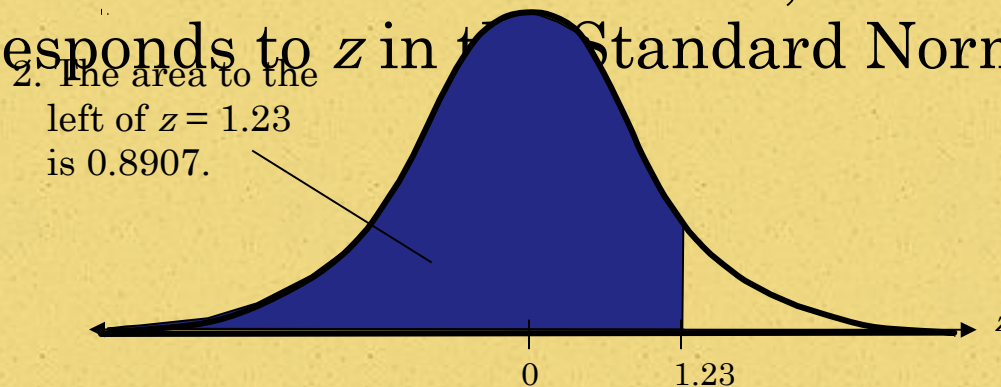| z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
−0.0	.4641	.4681	.4724	.4761	.4801	.4840	.4880	.4920	.4960	.5000

Find the area by finding −0.2 in the left hand column, and then moving across the row to the column under 0.05.

The area to the left of $z = -0.25$ is 0.4013
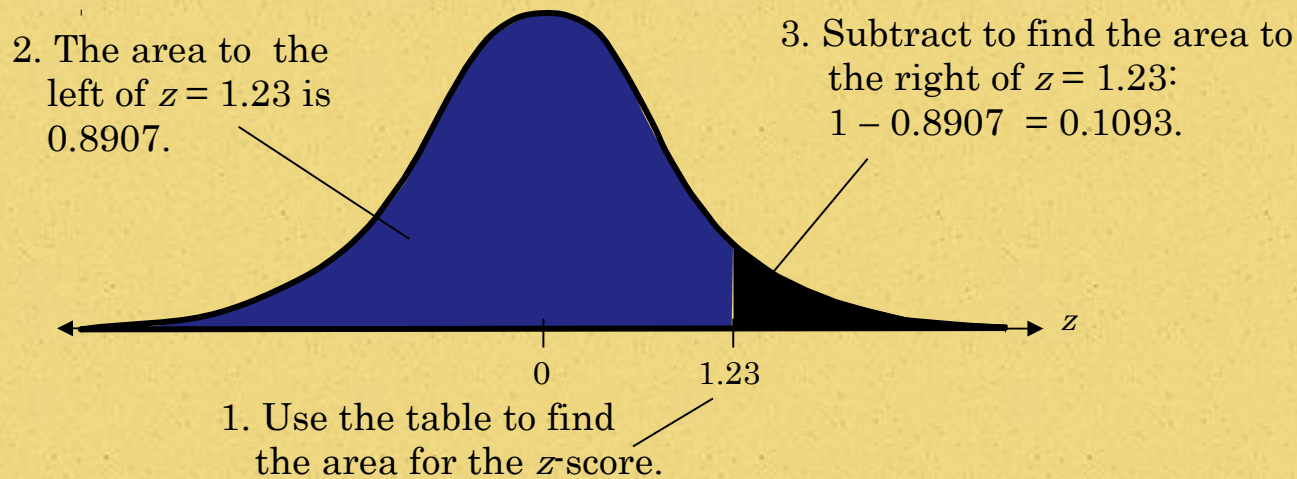
Finding Areas Under the Standard Normal Curve

1. Sketch the standard normal curve and shade the appropriate area under the curve.

2. Find the area by following the directions for each case shown.

   a. To find the area to the *left* of *z*, find the area that corresponds to *z* in the Standard Normal Table.

2. The area to the left of *z* = 1.23 is 0.8907.

1. Use the table to find the area for the *z*-score.

0   1.23
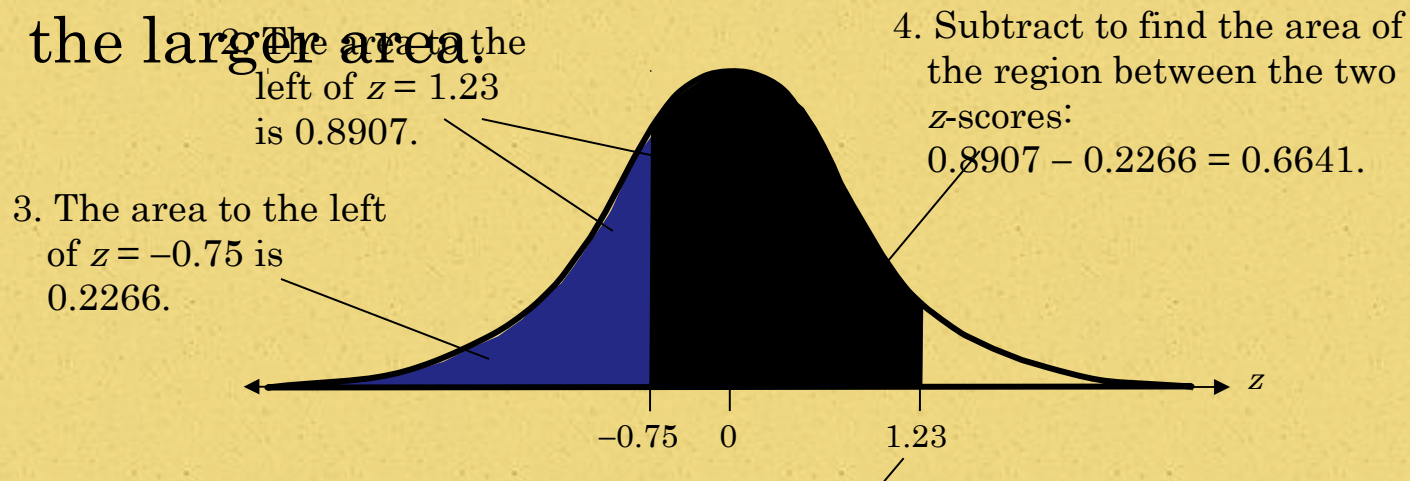
z

Finding Areas Under the Standard Normal Curve

b. To find the area to the *right* of *z*, use the Standard Normal Table to find the area that corresponds to *z*. Then subtract the area from 1.

2. The area to the left of $z = 1.23$ is 0.8907.

3. Subtract to find the area to the right of $z = 1.23$: $1 - 0.8907 = 0.1093$.

0   1.23

z

1. Use the table to find the area for the *z*-score.

Guidelines for Finding Areas

Finding Areas Under the Standard Normal Curve

c. To find the area *between* two *z*-scores, find the area corresponding to each *z*-score in the Standard Normal Table. Then subtract the smaller area from the larger area.
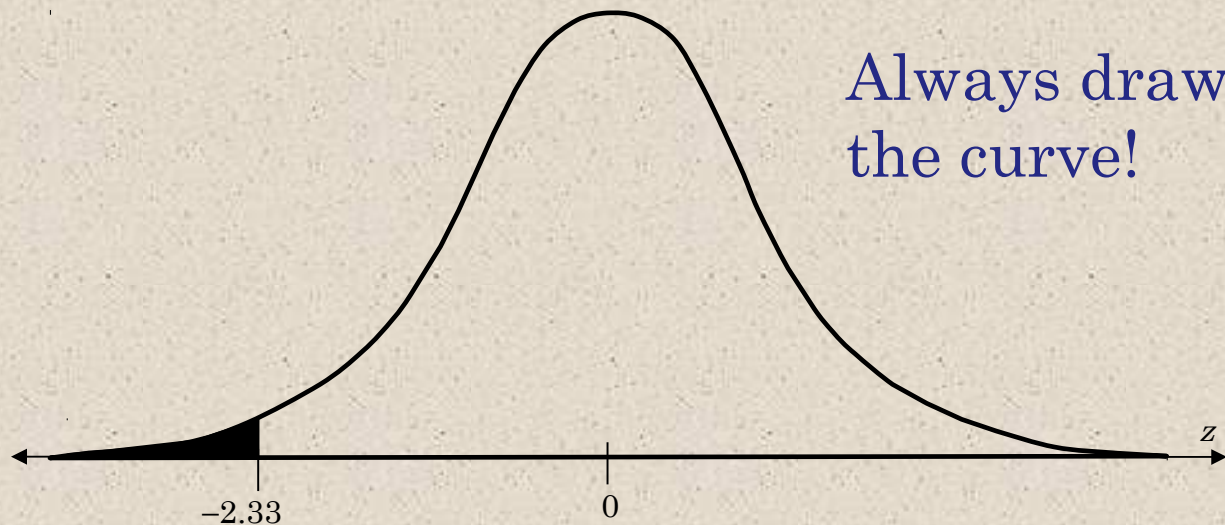
2. The area to the left of $z = 1.23$ is 0.8907.

4. Subtract to find the area of the region between the two *z*-scores: $0.8907 - 0.2266 = 0.6641$.

3. The area to the left of $z = -0.75$ is 0.2266.

−0.75   0   1.23

1. Use the table to find the area for the *z*-score.

Example:

Find the area under the standard normal curve to the left of $z = -2.33$.
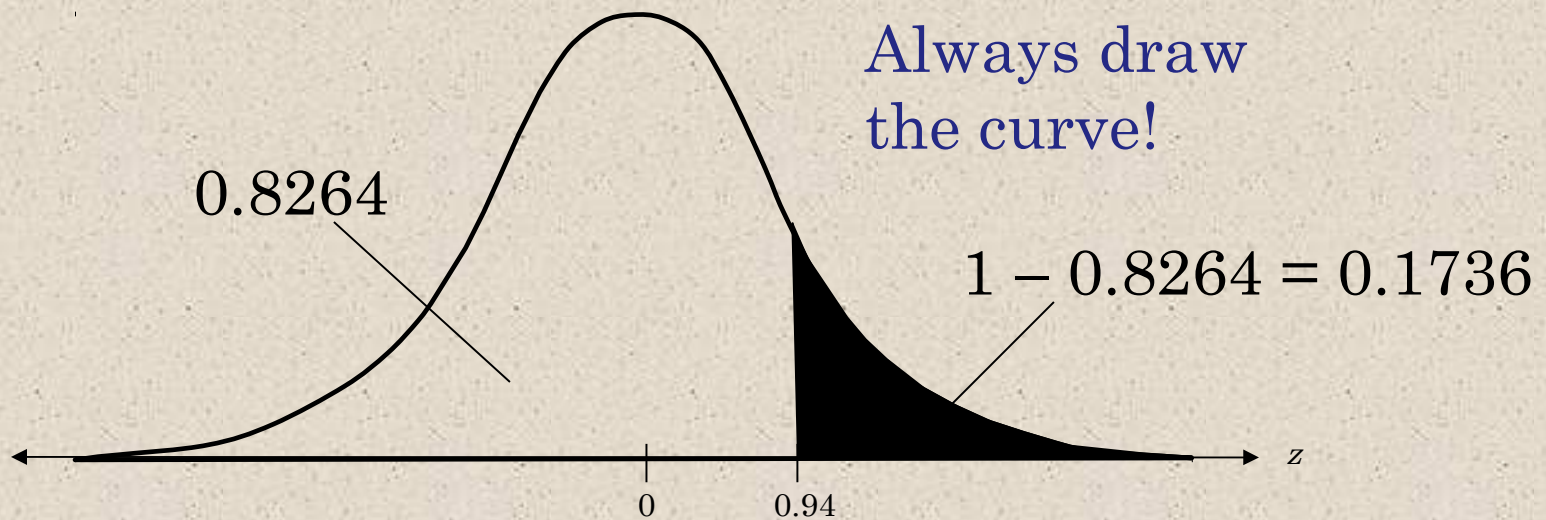

Always draw the curve!

−2.33   0

From the Standard Normal Table, the area is equal to 0.0099.

Guidelines for Finding Areas

Example:
Find the area under the standard normal curve to the right of $z = 0.94$.

Always draw the curve!
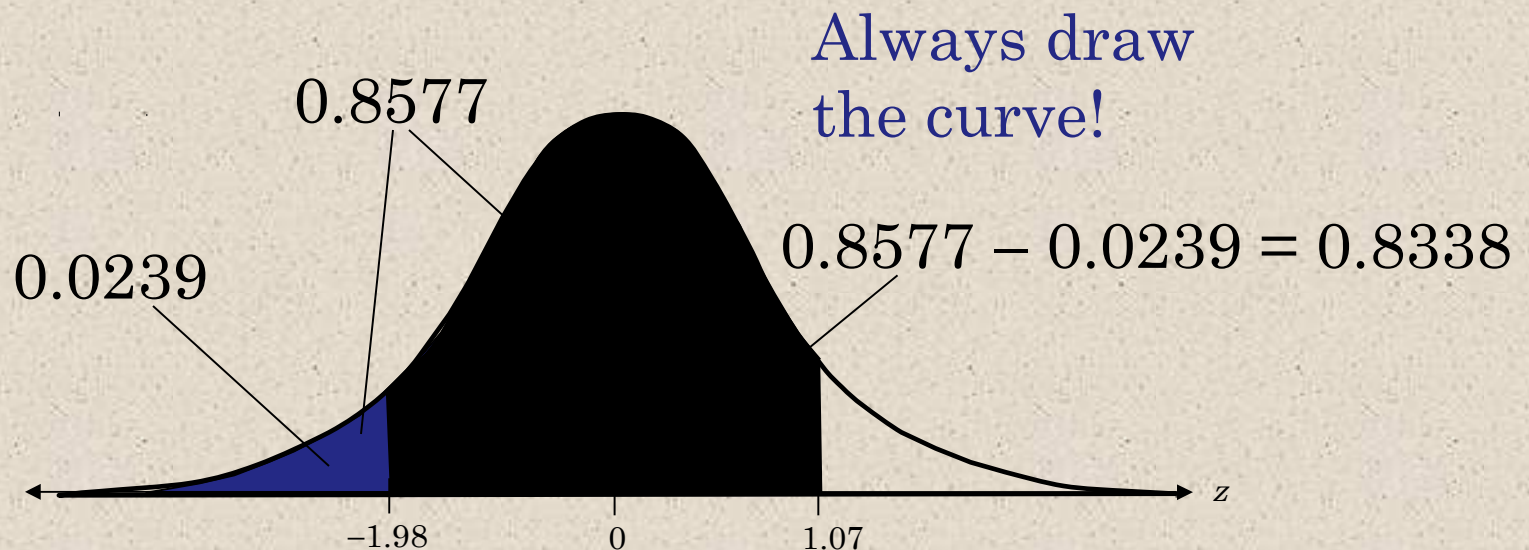
0.8264

$1 - 0.8264 = 0.1736$

0

0.94

z

From the Standard Normal Table, the area is equal to 0.1736.

Guidelines for Finding Areas

Example:
Find the area under the standard normal curve between $z = -1.98$ and $z = 1.07$.

Always draw the curve!

0.8577

0.0239

$0.8577 - 0.0239 = 0.8338$

−1.98   0   1.07 z

From the Standard Normal Table, the area is equal to 0.8338.

By using the z-score formula

Find the z-score then the area under the curve using your Standard Normal Table to find the probability.

In a recent year, the ACT scores for the English portion of the test were normally distributed, with a mean of 24.2 and a standard deviation of 2.7. A high school student who took the English portion of the ACT is randomly selected.

1. Find the probability that the student's ACT score is less than 19.

2. Find the probability that the student's ACT score is between 17 and 26.

3. Find the probability that the student's ACT score is more than 30.

Finding z-scores when the area is known.

3 **Find the z-score that corresponds to the given cumulative area by using your Standard Normal Table.**

1. 0.94

2. 0.4364

Find the z-score when the Percentile is known

Find the z-score that corresponds to the given percentile.

1. P7

2. P76

1. Find the z-score that has 17.9% of the distribution's area to its left.

2. Find the z-score that has 62.5% of the distribution's area to its right.

In a survey of men in the United States (ages 20-29)

The mean height was 69.9 inches with a standard deviation of 3.0 inches. **First find the z-score by using the Standard Normal Table, then by using the formula , solve for the x value.**

1. What height represents the 90th percentile?

2. What height represents the first quartile?

Sample mean and standard deviation

A population has a mean of 10 and a standard deviation of 2. Find the mean and standard deviation of a sample distribution with the following sample size.

36. n=100

37. n=250

Sample Word Problem

 The per capita consumption of coffee by people in the United states in a recent year was normally distributed with a mean of 26 gallons and a standard deviation of 5 gallons. Random samples of 25 are drawn from this population and the mean of each sample is determined.

1. Find the sample mean.

2. Find the standard error of the mean.